Learning From Data

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Lecture 4: Error and Noise

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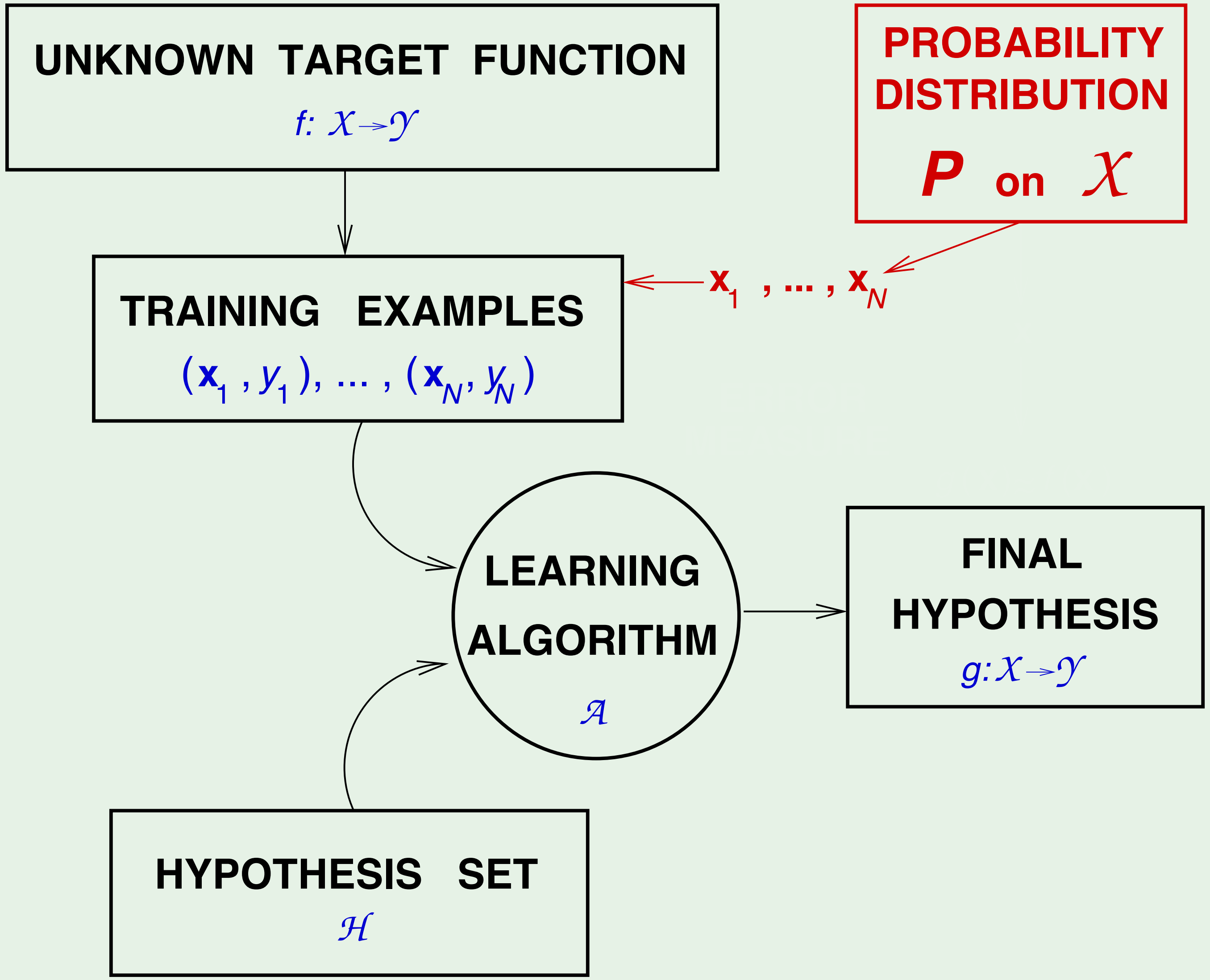
The learning diagram - where we left it



Error measures

What does "$h \approx f$" mean?

Error measure: $E(h, f)$

Almost always *pointwise definition*: $\mathrm{e}\left(h(\mathbf{x}), f(\mathbf{x})\right)$

Examples:

Squared error: $\mathrm{e}\left(h(\mathbf{x}), f(\mathbf{x})\right) = \left(h(\mathbf{x}) - f(\mathbf{x})\right)^2$

Binary error: $\mathrm{e}\left(h(\mathbf{x}), f(\mathbf{x})\right) = [\![h(\mathbf{x}) \neq f(\mathbf{x})]\!]$

From pointwise to overall

Overall error $E(h, f)$ = average of pointwise errors $\mathrm{e}\left(h(\mathbf{x}), f(\mathbf{x})\right)$.

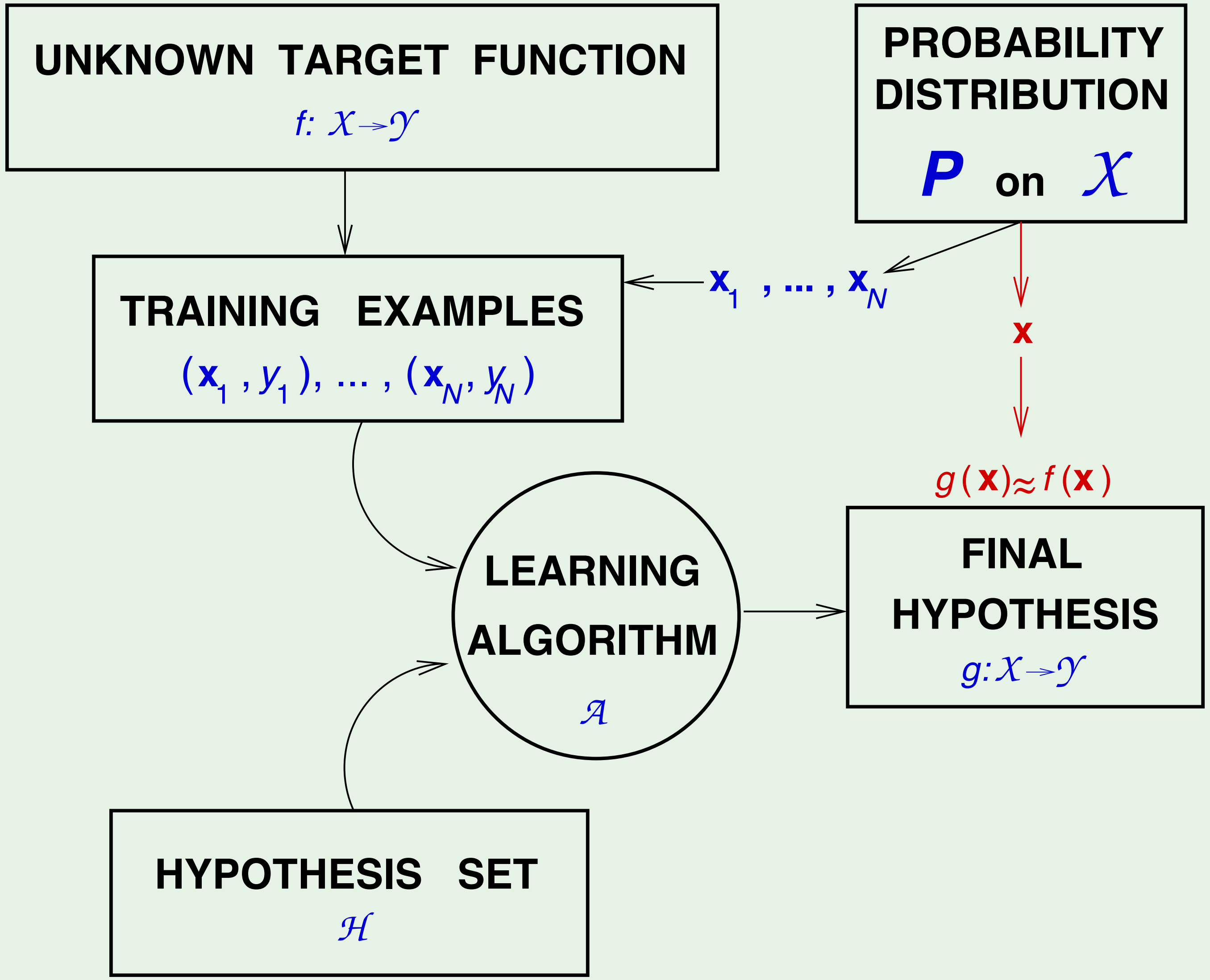
In-sample error:

$$E_{\text{in}}(h) = \frac{1}{N} \sum_{n=1}^{N} \mathrm{e}\left(h(\mathbf{x}_n), f(\mathbf{x}_n)\right)$$

Out-of-sample error:

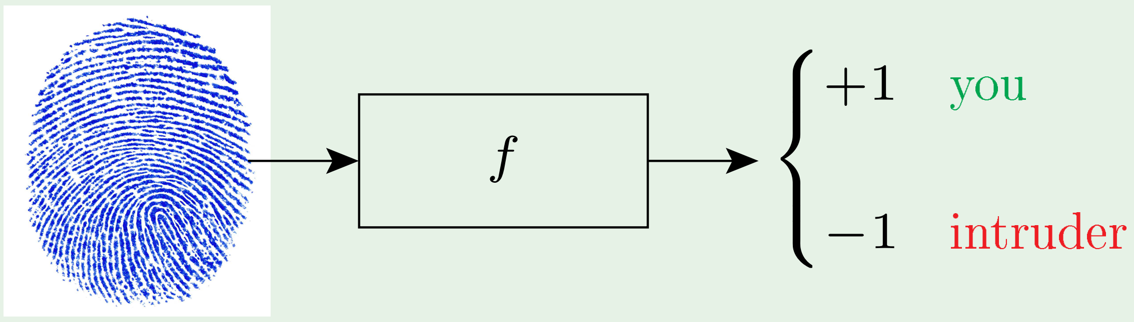
$$E_{\text{out}}(h) = \mathbb{E}_{\mathbf{x}}\left[\mathrm{e}\left(h(\mathbf{x}), f(\mathbf{x})\right)\right]$$

The learning diagram - with pointwise error



How to choose the error measure

Fingerprint verification:



Two types of error:

false accept and *false reject*

How do we penalize each type?

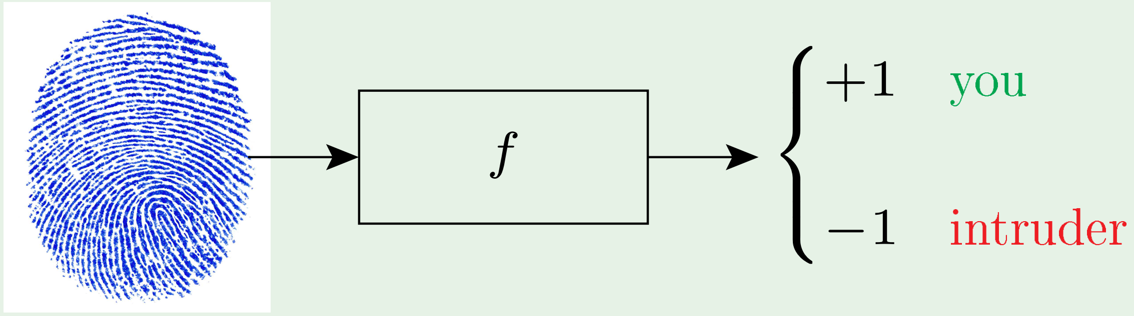
		f: $+1$	f: -1
h	$+1$	no error	*false accept*
	-1	*false reject*	no error

The error measure - for supermarkets

Supermarket verifies fingerprint for discounts

False reject is costly; customer gets annoyed!

False accept is minor; gave away a discount and intruder left their fingerprint ☺



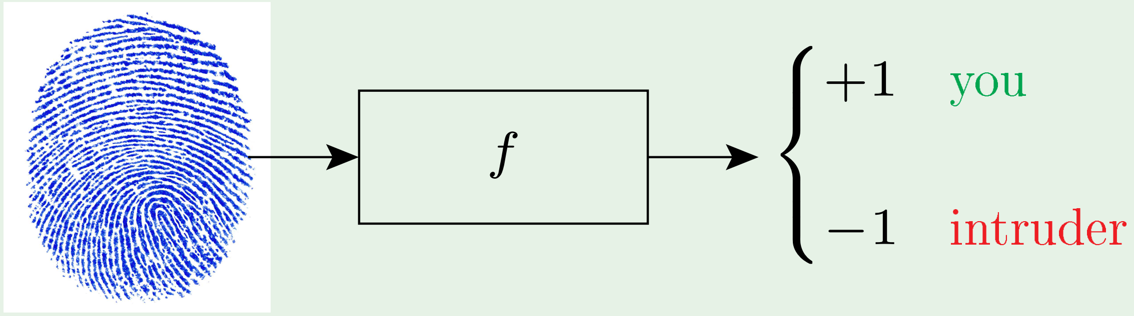
		f: $+1$	f: -1
h	$+1$	0	1
	-1	10	0

The error measure - for the CIA

CIA verifies fingerprint for security

False accept is a disaster!

False reject can be tolerated
Try again; you are an employee ☺



		f = +1	f = −1
h	+1	0	1000
h	−1	1	0

Take-home lesson

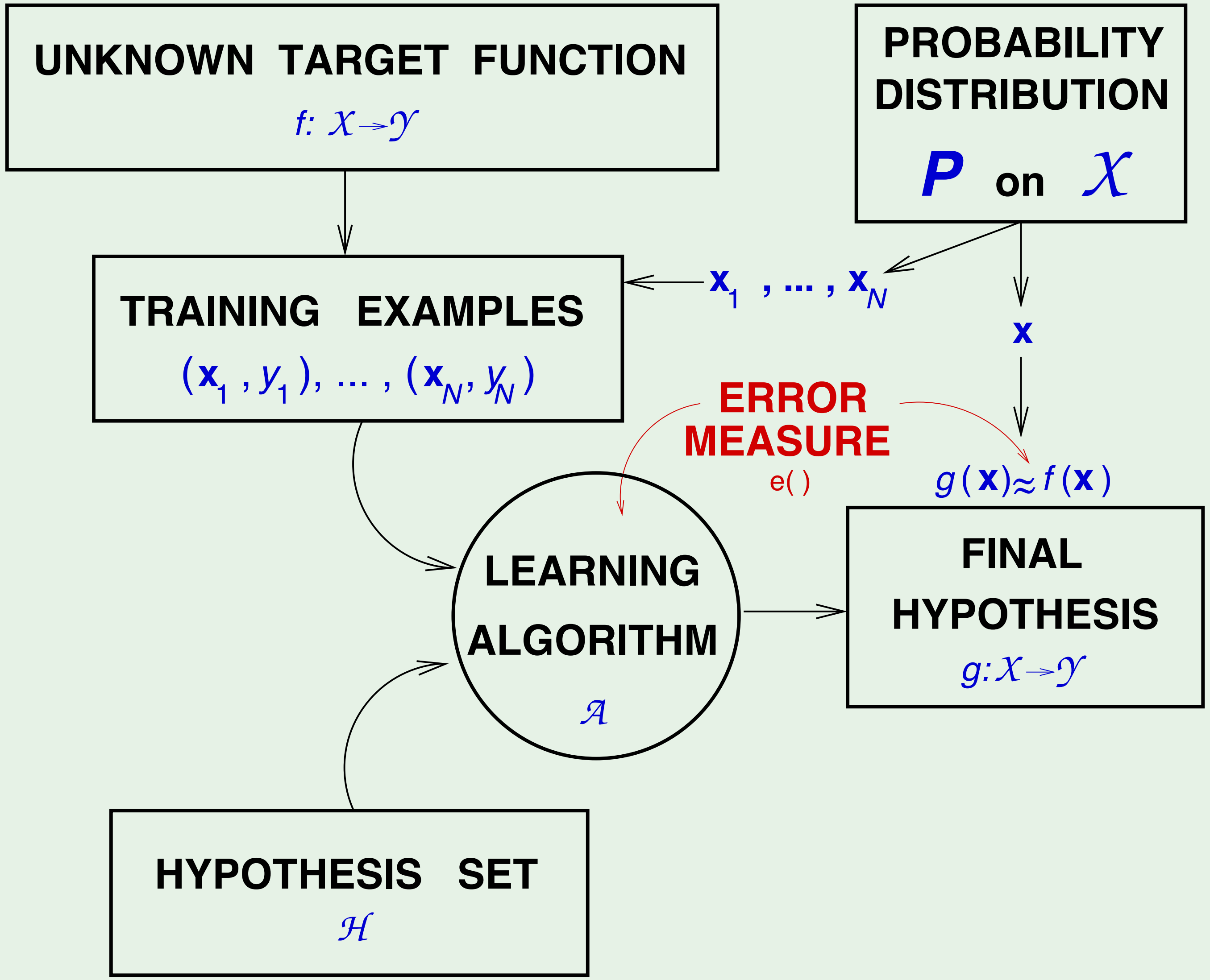
The error measure should be specified by the user.

Not always possible. Alternatives:

Plausible measures: squared error $\equiv$ Gaussian noise

Friendly measures: closed-form solution, convex optimization

The learning diagram - with error measure



Noisy targets

The 'target function' is not always a *function*

Consider the credit-card approval:

age	23 years
annual salary	\$30,000
years in residence	1 year
years in job	1 year
current debt	\$15,000
⋯	⋯

two 'identical' customers $\longrightarrow$ two different behaviors

Target 'distribution'

Instead of $y = f(\mathbf{x})$, we use target *distribution*:

$$P(y \mid \mathbf{x})$$

$(\mathbf{x}, y)$ is now generated by the joint distribution:

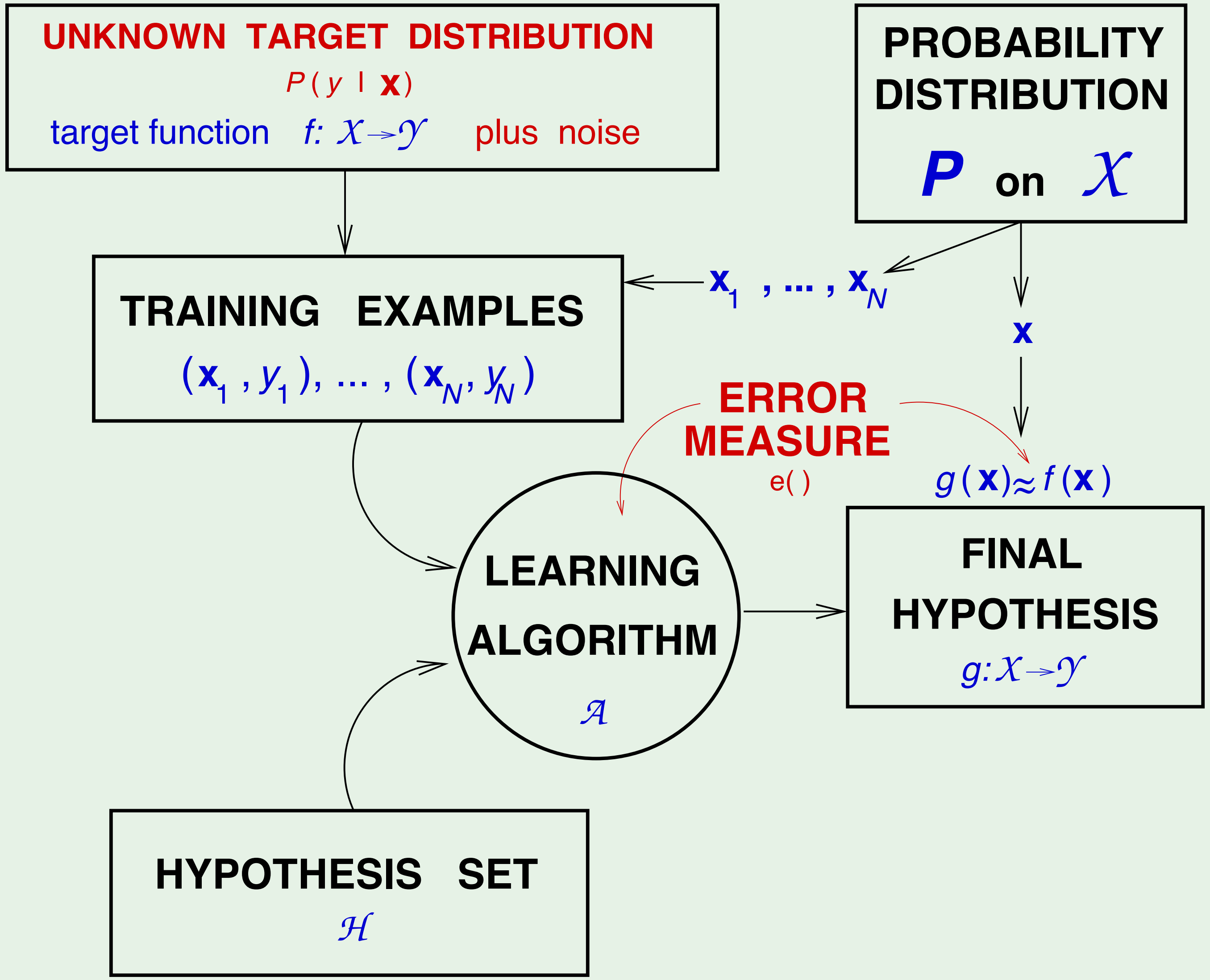
$$P(\mathbf{x})P(y \mid \mathbf{x})$$

Noisy target = deterministic target $f(\mathbf{x}) = \mathbb{E}(y|\mathbf{x})$ plus noise $y - f(\mathbf{x})$

Deterministic target is a special case of noisy target:

$$P(y \mid \mathbf{x}) \text{ is zero except for } y = f(\mathbf{x})$$

The learning diagram - including noisy target

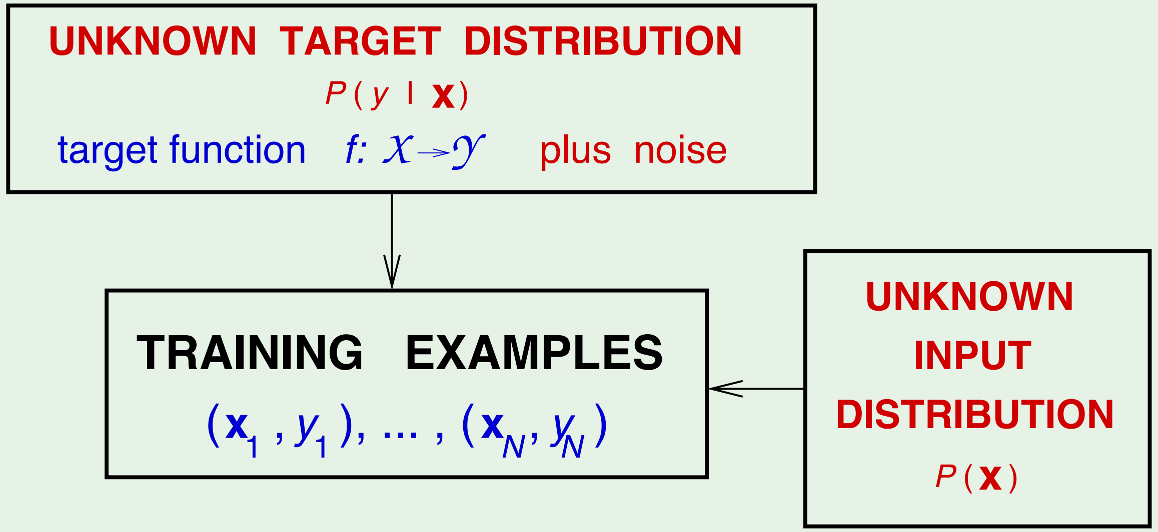


Distinction between $P(y|\mathbf{x})$ and $P(\mathbf{x})$

Both convey probabilistic aspects of $\mathbf{x}$ and y

The target distribution $P(y \mid \mathbf{x})$

is what we are trying to learn



The input distribution $P(\mathbf{x})$

quantifies relative importance of $\mathbf{x}$

Merging $P(\mathbf{x})P(y|\mathbf{x})$ as $P(\mathbf{x}, y)$

mixes the two concepts

What we know so far

Learning is feasible. It is likely that

$$E_{\text{out}}(g) \approx E_{\text{in}}(g)$$

Is this learning?

We need $g \approx f$, which means

$$E_{\text{out}}(g) \approx 0$$

What the theory will achieve

Characterizing the feasibility of learning for infinite M

Characterizing the tradeoff:

Model complexity	$\uparrow$	E_{in}	$\downarrow$
Model complexity	$\uparrow$	$E_{\text{out}} - E_{\text{in}}$	$\uparrow$